PERFECT NUMBER

THE PREMIER DSE MOCK EXAM

C. F. Tang

Perfect Number | Sample Mock Examination

Mathematics (Compulsory Part)

Paper 2

Time allowed : 1 hour 15 minutes

Full mark : 45

This question book consists of 13 printed pages.

Instructions to candidates:

1. This paper consists of 45 multiple-choice questions.
   There are 30 questions in Section A and 15 questions in Section B. All questions carry equal marks.

2. Answer ALL questions.

3. Choose the best answer for each question.

4. When told to check the question paper, you should check that all the questions are there. Look for the words 'END OF PAPER' after the last question.

5. You should mark only ONE answer for each question. If you mark more than one answer, you will receive NO MARKS for that question.

6. No marks will be deducted for wrong answers.

7. The diagrams in this paper are not necessarily drawn to scale.

8. Calculator pad printed with the "HKEA Approved" / "HKEAA Approved" label is allowed.

There are 30 questions in Section A and 15 questions in Section B.

The diagrams in this paper are not necessarily drawn to scale.

Choose the best answer for each question.

Section A

1. $(-8)^{2n-1} \cdot (-2)^{2n+1} =$

   A. 2^{-8n+2}

   B. 4^{4n-1}

   C. -4^{4n-1}

   D. -4^{-4n+1}

2. for $x > 1$, if $x + \frac{1}{x} = k$, then $x^2 - \frac{1}{x^2} =$

   A. $k\sqrt{k^2 - 4}$

   B. $k\sqrt{k^2 - 2}$

   C. $k^2 - 4$

   D. $k^2 - 2$

3. If $2x = 3y = 4z$, then $\frac{x+y}{z} =$

   A. $\frac{3}{10}$

   B. $\frac{4}{5}$

   C. $\frac{5}{4}$

   D. $\frac{10}{3}$

4. If $A(x+1)(x-2) + B(x+1)(x+3) + C(x-2)(x+3) \equiv 5x^2 + 12x - 18$. Find the value of $2A + B + C$.

   A. $-\frac{12}{5}$

   B. $\frac{41}{10}$

   C. 5

   D. $\frac{31}{5}$

5. If $\alpha,\ \beta$ are roots of $2x^2+3x-7=0$, then $2\alpha^2-3\beta=$

A. 10

B. 11

C. $\frac{23}{2}$

D. $\frac{25}{2}$

6. Let $f(x+1)=2x^3-5x+8$, find the remainder when $f(x)$ is divided by $x+1$.

A. -2

B. 2

C. 5

D. 10

7. If $f(2x+1)=\frac{3+2x}{2-3x}$, find $f(x)$.

A. $\frac{2+x}{3-4x}$

B. $\frac{2x}{x+1}$

C. $\frac{2+x}{3-2x}$

D. $\frac{4+2x}{7-3x}$

8. The speed of a train is decreased by r %. Find the percentage change in the time taken to travel the same distance.

A. $\frac{100r}{1-r}$ %

B. $\frac{r}{r-100}$ %

C. $\frac{100r}{100-r}$ %

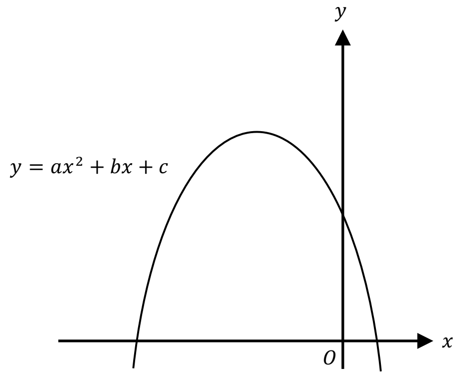
D. $\frac{r}{1-100r}$ %

9. The figure shows the graph of $y = ax^2 + bx + c$. Which of the following is/are true?

I. $a < 0$

II. $b > 0$

III. $\frac{b}{c} < \frac{4a}{b}$



A. I and II only

B. II and III only

C. I and II only

D. I and III only

10. The coordinates of point A are $(3, -4)$. If A is rotated anticlockwise about the origin through 270^o and then reflected along the x-axis to point B, find the coordinates of point B.

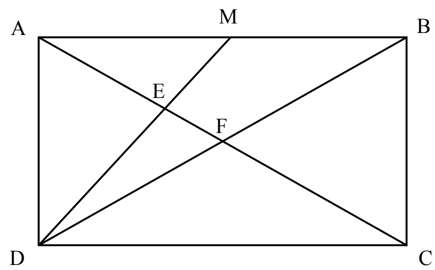
A. $(-3, -4)$

B. $(-4, 3)$

C. $(-4, -3)$

D. $(3, 4)$

11. In the figure, $ABCD$ is a rectangle. M is the mid-point of AB. DM and DB intersects AC at E and F respectively. Find the ratio of area of ΔDEF to ΔBCF.



A. $1 : 4$

B. $1 : 3$

C. $2 : 5$

D. $3 : 7$

12. The equations of straight lines L_1 and L_2 are $y = 2x$ and $y = -x$ respectively. If P is a moving point such that P is equidistant from L_1 and L_2, the locus of P is/are

A. a straight line which is parallel to L_1 and L_2

B. a parabola

C. a circle with origin as centre

D. a pair of straight lines that pass through the origin

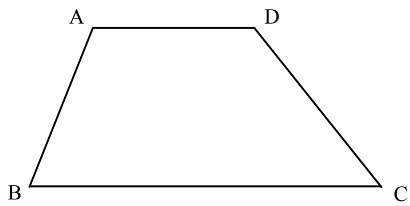
13. In the figure, $ABCD$ is a trapezium with $AD//BC$. $AB = 4$, $BC = 8$, $AD = 5$, $\angle BAD = 115^o$. Find CD correct to 1 decimal place.

A. 3.8

B. 3.9

C. 4.2

D. 4.3



14. When $x + 2x^2 + 3x^3 + 4x^4 + \cdots + 2nx^{2n}$, where n is a positive integer, is divided by $x + 1$, the remainder is

A. $-n$

B. n

C. $-\frac{n}{2}$

D. $\frac{n}{2}$

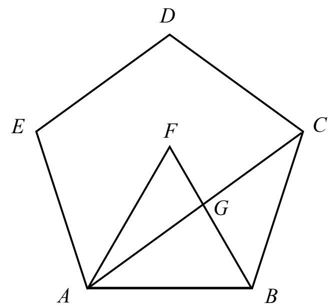
15. In the figure, $ABCDE$ is a regular pentagon and ΔABF is an equilateral triangle. If AC meets FB at G, then $\angle AGB =$

A. 82^o

B. 84^o

C. 92^o

D. 96^o



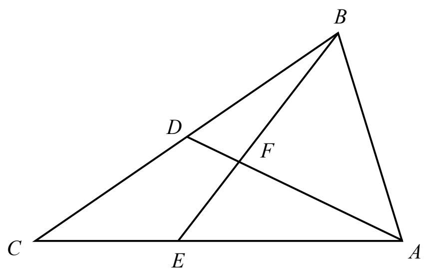
16. In the figure, AD is a median of ΔABC and E is a point lying on AC such that $AE = 3EC$. BE intersects AD at F. The ratio of $BF : FE =$

A. $1 : 1$

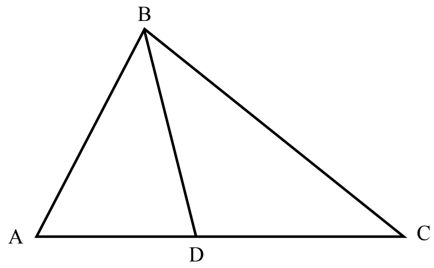
B. $3 : 2$

C. $5 : 4$

D. $4 : 3$



17. In the figure, BD is the angle bisector of $\angle ABC$. Which of the following is correct?



A. $AB \times BC = CD \times AD$

B. $AB \times AD = BC \times CD$

C. $AB \times BD = BC \times AD$

D. $AB \times CD = BC \times AD$

18. $\dfrac{\sin^2(180°+\theta)+\cos^2\theta}{\tan(90°+\theta)\cos(180°+\theta)}$

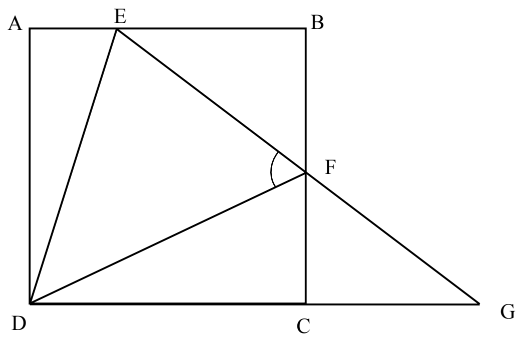
A. $\dfrac{\sin^2\theta}{\cos\theta}$

B. $\dfrac{\sin\theta}{\cos^2\theta}$

C. $\dfrac{\cos^2\theta}{\sin\theta}$

D. $\dfrac{\cos\theta}{\sin^2\theta}$

19. In the figure, $ABCD$ is a square, E is a point divided AB into $1:2$, F is a point on BC such that EF produced meet DC produced at G. If $DG = EG$, find $\angle DFE$ correct to nearest degree.



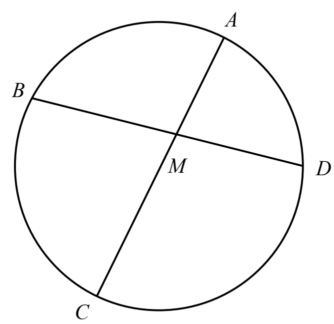
A. 55^o

B. 58^o

C. 60^o

D. 63^o

20. In the figure, $BM = 4\ cm$, $CM = 6\ cm$, $DM = 5\ cm$, $AM =$



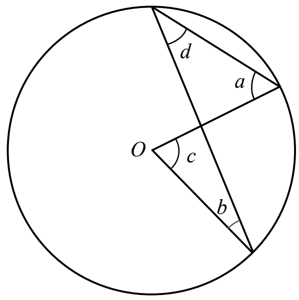
A. $\dfrac{10}{3}$

B. $\dfrac{24}{5}$

C. $\dfrac{15}{2}$

D. Can't be determined

21. In the figure, O is the centre of the circle. a, b, c and d are angles at circumference. Which of the following is/are correct?



I. $a = 2b$

II. $c = 2d$

III. $2a = c + 2b$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

22. $\frac{1+bi^3}{i} =$

A. $b + i$

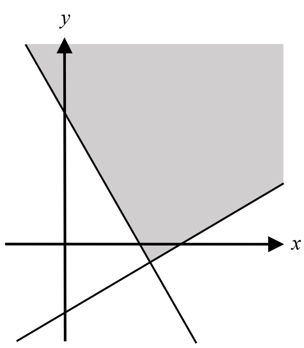
B. $b - i$

C. $-b + i$

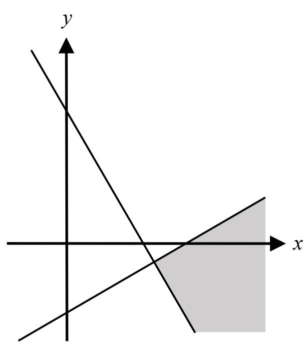
D. $-b - i$

23. Which of the following regions may represent the solution of $\begin{cases} 2x + y - 5 \geq 0 \\ x - y - 3 \geq 0 \end{cases}$

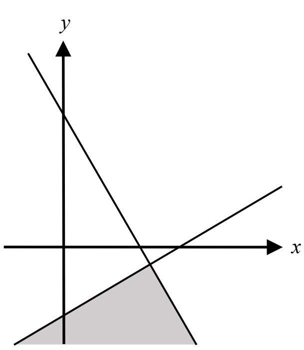
A.



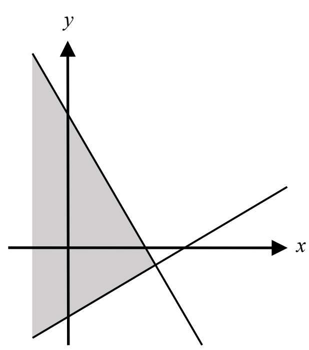
B.



C.



D.



24. Peter purchase a box of N oranges with \$ x , afterward he found that 10 of them are rotten, then he sold the rest at a price which \$ 3 more than the cost price of each orange, at a result he earned \$ 100. Which of the following relation between N and x is correct?

A. $(N-10)(x+3)=100$

B. $(N-10)(\frac{x}{N}+3) = 100$

C. $(N-10)(x+3) = x+100$

D. $(N-10)(\frac{x}{N}+3) = x+100$

25. Suppose that a teacher has had 4 of his students, A, B, C, and D, take a test and wants to let them grade each other's. How many ways could the teacher hand the tests back to the students for grading, such that no student received his or her own test back?

A. 3

B. 6

C. 9

D. 23

26. There are six cards numbered 0, 1, 2, 2, 3, 4. Two cards are chosen at random, find the probability that the product of two numbers is an even number.

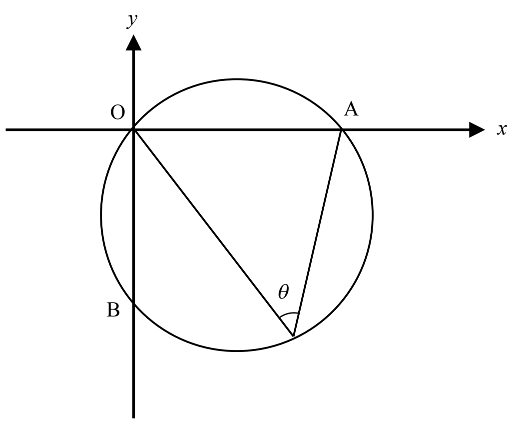
A. $\frac{1}{15}$

B. $\frac{2}{5}$

C. $\frac{8}{15}$

D. $\frac{14}{15}$

27. In the figure, the circle $x^2+y^2-4x+6y=0$ cut x-axis and y-axis at A and B respectively, find the value of $\tan\theta$.



A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

28. The following table shows the number of books rented from a bookstore by 80 customers.

Number of books rented	0	1	2	3	4
Frequency	24	19	17	11	9

Find the inter-quartile range of the books rented.

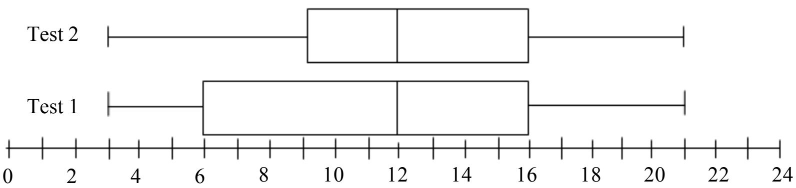
A. 2

B. 2.5

C. 3

D. 4

29. The box-and-whisker diagram below shows the distribution of the score of two tests from same group of students. Which of the following must be false?



A. mean score of test 1 < mean score of test 2

B. median score of test 1 = median score of test 2

C. the distribution of test 2 is less dispersed than test 1

D. the number of students in test 2 who score less than 9 is more than that in test 1

30. What is the domain of the function $y = \frac{2}{\sqrt{x-1}} + \sqrt{3-x}$

A. $1 < x \leq 3$

B. $1 \leq x \leq 3$

C. $x \leq 3$

D. $x > 1$

Section B

31. If a straight line $kx + y = 1$ cuts the curve $y = x^2$ at A and B, find in term of k, the coordinates of mid-point of A and B.

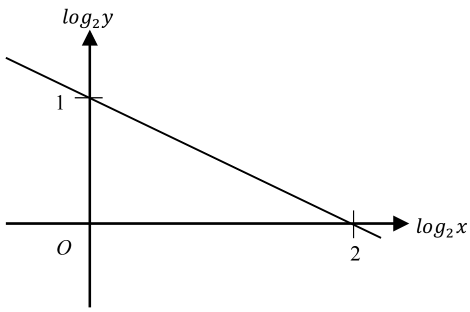
A. $(-\frac{k}{2}, \frac{2+k^2}{2})$

B. $(-\frac{k}{2}, \frac{k^2}{4})$

C. $(0, 1)$

D. $(-\frac{k}{2}, 1)$

32. The graph in the figure shows the linear relation between $log_2 x$ and $log_2 y$. Which of the following must be true?



A. $xy^2 = 4$

B. $x^2 y = 4$

C. $y = x^{-\frac{1}{2}} + 2$

D. $y = x^{-2} + 2$

33. $29 \times 16^8 + 4 \times 16^3 + 12 \times 16^2 =$

A. $1D00004C00_{16}$

B. $1D00004C0_{16}$

C. $1D0004C00_{16}$

D. $1D0004C0_{16}$

34. If α, β are roots of $ax^2 + bx + c = 0$, then $\alpha^4 + \beta^4 =$

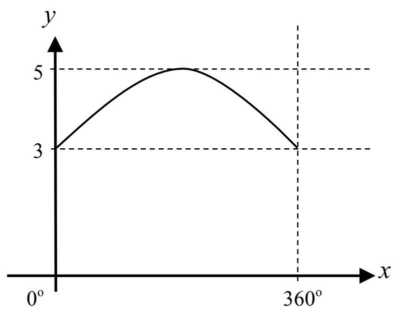
A. $\frac{b^4 - 4ab^2c + 2a^2c^2}{a^4}$

B. $\frac{b^4 - 4ab^2c + 4a^2c^2}{a^4}$

C. $\frac{b^4 + 4ab^2c + 2a^2c^2}{a^4}$

D. $\frac{b^4 + 4ab^2c + 4a^2c^2}{a^4}$

35. The figure shows the graph of



A. $y = 3 + \sin 2x$

B. $y = 3 + 2\sin\frac{x}{2}$

C. $y = 3 - \sin 2x$

D. $y = 3 - 2\sin\frac{x}{2}$

36. If $log_2(x + 1) - log_2(2x + 1) < 0$, then

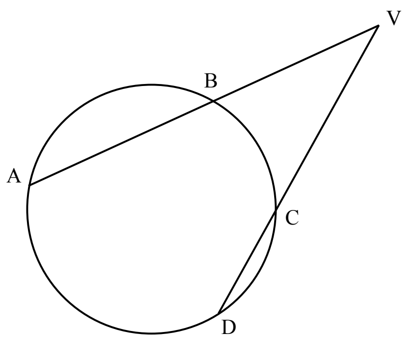
A. $x > 0$

B. $x < -\frac{1}{2}$ or $x > 0$

C. $x < -1$ or $x > -\frac{1}{2}$

D. $x < -1$

37. In the figure, A and D are points on the circle, VA and VD cut the circle at B and C respectively. If $AB = BV = 12$, $VC = 16$, find CD.



A. 2

B. 8

C. 12

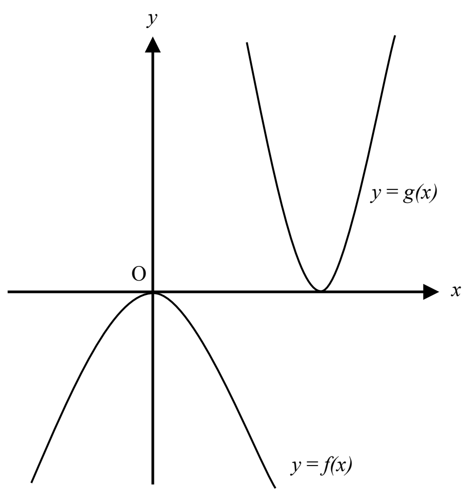
D. 16

38. The figure shows the graphs of $y = f(x)$ and $y = g(x)$. $f(x)$ is a quadratic function having vertex at the origin. The graph of $y = g(x)$ can be obtained by transformation of the graph of $y = f(x)$. If $a > 0$, which of the following may be the relation between $f(x)$ and $g(x)$?

I. $g(x) = -f(2x - a)$

II. $g(x) = -a\,f(x - a)$

III. $g(x) = -2\,f(2x - a)$



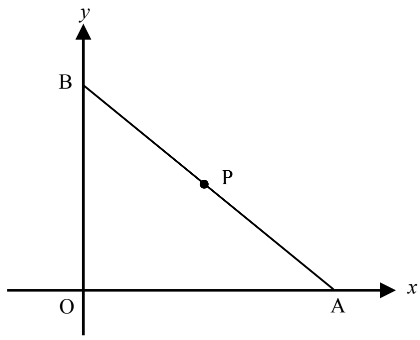
A. I only

B. II only

C. I and III only

D. I, II and III

39. A, B are the point on x-axis and y-axis respectively. P is the mid-point of AB such that $AB = 10$, what is the equation of locus of P?



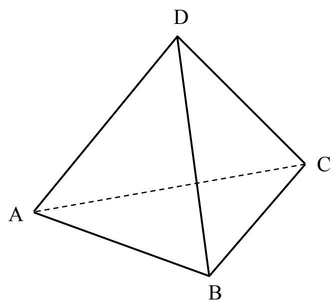
A. $y = x$

B. $x^2 + y^2 = 25$

C. $x^2 + y^2 = 100$

D. $x^2 + y^2 - 20x - 20y + 100 = 0$

40. The figure shows a regular tetrahedron with ΔABC as its base, if the area of ΔABC is $\sqrt{3}\ cm^2$, what is the volume of the solid?



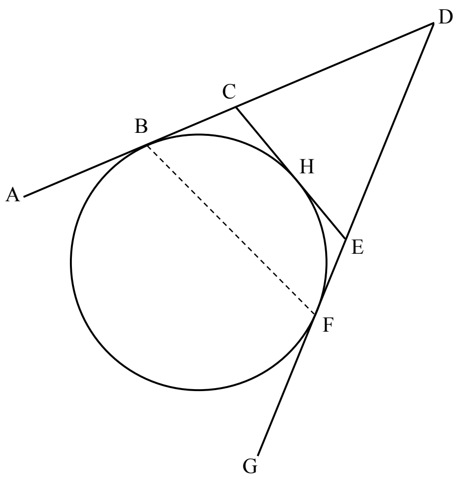
A. $\frac{\sqrt{3}}{6}\ cm^3$

B. $\frac{\sqrt{3}}{3}\ cm^3$

C. $\frac{\sqrt{6}}{6}\ cm^3$

D. $\frac{2\sqrt{2}}{3}\ cm^3$

41. In the figure, AD, GD and CE are tangents to the circle at B, F and H respectively. Which of the following must be correct?



I. $CH + CD = DF$

II. $\angle DCE = \angle DEC$

III. $\angle CBF = \angle CED$

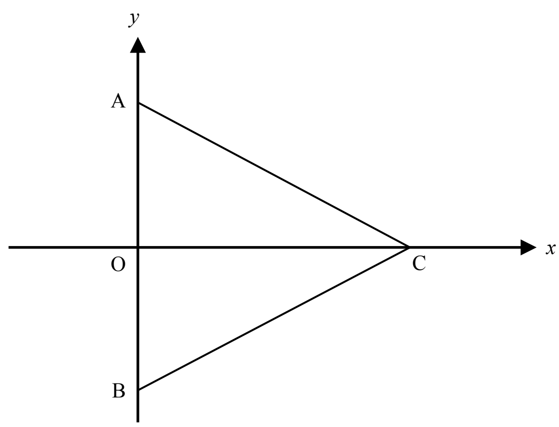
A. I only

B. I and II only

C. I and III only

D. II and III only

42. In the figure, the coordinates of A, B and C are (0, 3), (0, –3) and (4, 0) respectively, find the coordinates of the circumcentre of ΔABC.



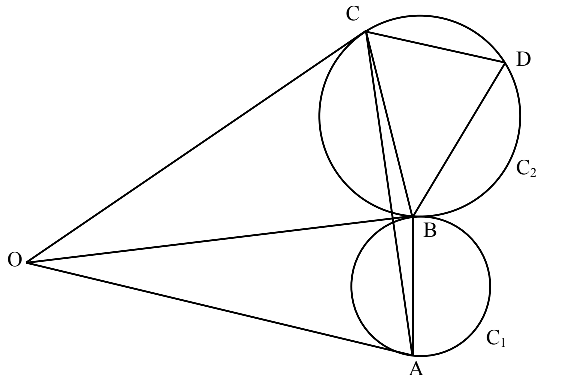
A. $(\frac{4}{9}, 0)$

B. $(\frac{7}{8}, 0)$

C. $(\frac{4}{3}, 0)$

D. $(\frac{8}{3}, 0)$

43. In the figure, two circle C_1 and C_2 touch externally at B such that OB is their common tangent, OA and OC are two other tangents to C_1 and C_2 respectively. D is a point on C_2 such that $\angle AOB : \angle BOC : \angle BDC = 1:2:3$, find $\angle ACB$.



A. 7.5^o

B. 10.25^o

C. 11.25^o

D. 12.5^o

44. If *a*, *b*, *c* is a geometric sequence, which of the following must be true?

I. $\log\sqrt{a}$, $\log\sqrt{b}$, $\log\sqrt{c}$ is an arithmetic sequence.

II. $2^{\log a^2}$, $2^{\log b^2}$, $2^{\log c^2}$ is a geometric sequence.

III. $ax^2 + 2bx + c = 0$ has double roots.

A. I and II only

B. II and III only

C. I and III only

D. I, II and III

45. There are two group of students with mean and variance of their height as follow,

Group	Number of students	Mean(cm)	Variance(cm^2)
A	8	168	36
B	6	168	48

What is the variance of the height of these 14 students? Correct your answer to 1 decimal place.

A. 27.4 cm^2

B. 41.1 cm^2

C. 41.8 cm^2

D. 42.0 cm^2

END OF PAPER

SAMPLE MOCK EXAMINATION ANSWER AND SOLUTION

1.	B	11.	B	21.	C	31.	A	41.	A
2.	A	12.	D	22.	D	32.	A	42.	B
3.	D	13.	B	23.	B	33.	A	43.	C
4.	B	14.	B	24.	D	34.	A	44.	B
5.	C	15.	B	25.	C	35.	B	45.	B
6.	B	16.	D	26.	D	36.	A		
7.	D	17.	D	27.	C	37.	A		
8.	C	18.	B	28.	B	38.	C		
9.	D	19.	D	29.	D	39.	B		
10.	B	20.	A	30.	A	40.	D		

1. $(-8)^{2n-1} \cdot (-2)^{2n+1}$
$=(-2)^{3(2n-1)} \cdot (-2)^{2n+1}$
$=(-2)^{6n-3+2n+1}$
$=(-2)^{8n-2}$
$=(2)^{2(4n-1)}$
$=4^{4n-1}$

2. $x+\frac{1}{x}=k$
$(x+\frac{1}{x})^2=k^2$
$x^2+2+\frac{1}{x^2}=k^2$
$x^2-2+\frac{1}{x^2}=k^2-4$
$(x-\frac{1}{x})^2=k^2-4$
$x-\frac{1}{x}=\sqrt{k^2-4}$
$\Rightarrow x^2-(\frac{1}{x})^2=(x+\frac{1}{x})(x-\frac{1}{x})$
$=k\sqrt{k^2-4}$

3. $2x=3y=4z$
$\Rightarrow x:y:z=\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$
$x:y:z=6:4:3$
$\therefore \frac{x+y}{z}=\frac{10}{3}$

4. $A(x+1)(x-2)+B(x+1)(x+3)+C(x-2)(x+3)\equiv 5x^2+12x-18$
sub $x=-3$
$10A=-9$
$A=-\frac{9}{10}$
by considering the coefficient of x^2,
$A+B+C=5$
$\therefore 2A+B+C=-\frac{9}{10}+5$
$=\frac{41}{10}$

5. $2x^2+3x-7=0$
$2\alpha^2+3\alpha-7=0$
$2\alpha^2=7-3\alpha$
$\therefore 2\alpha^2-3\beta$
$=7-3(\alpha+\beta)$
$=7-3\left(-\frac{3}{2}\right)=\frac{23}{2}$

6. remainder $=f(-1)$
$=f(-2+1)$
$=2(-2)^3-5(-2)+8$
$=2$

7. $f(x)=f(2\left(\frac{x-1}{2}\right)+1)$
$=\frac{3+2\left(\frac{x-1}{2}\right)}{2-3\left(\frac{x-1}{2}\right)}$
$=\frac{4+2x}{7-3x}$

8. Let s be speed, d be distance and t be time
$t=\frac{d}{s}$
$t_{new}=\frac{d}{(1-r\%)s}$
% change of speed $=\frac{\frac{d}{(1-r\%)s}-\frac{d}{s}}{\frac{d}{s}}\times 100\%$
$=[\frac{1}{(1-r\%)}-1]\times 100\%$
$=[\frac{r\%}{(1-r\%)}]\times 100\%$
$=\frac{100r}{100-r}\ \%$

9. concave downward, $a < 0$

$\because$ axis of asymptotes < 0

$\therefore -\frac{b}{2a} < 0$

$b < 0$

there are two x-intercepts

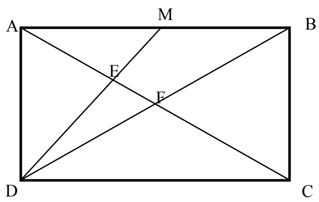
$b^2 - 4ac > 0$

$b^2 > 4ac$

$\frac{b}{c} < \frac{4a}{b}$ ($\because b < 0$ and $c > 0$)

10. $(3, -4) \Rightarrow (-4, -3) \Rightarrow (-4, 3)$

11.



$AE : EC = AM : FC = 1 : 2$

let $AE = 1, EF = 2$ and $EF = x$

$\because AF = FC$

$\therefore 1 + x = 2 - x$

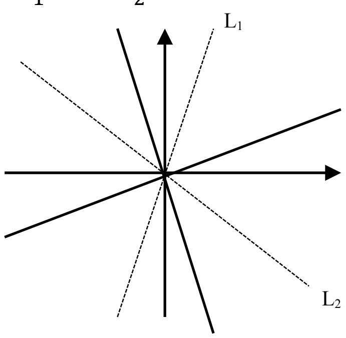
$x = 0.5$

$\Delta DEF : \Delta AED = 0.5 : 1 = 1 : 2$

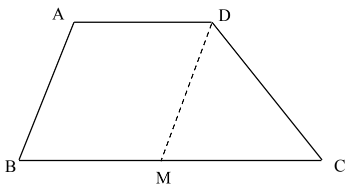
$\Delta ADF = \Delta BFC$

$\therefore \Delta DEF : \Delta BCF = 1 : 1 + 2 = 1 : 3$

12. The locus of P are the angle bisectors of L_1 and L_2



13. let M be a point on BC such that $DM//AB$



$CM = 8 - 5 = 3$

$DM = AB = 4$

$\angle DMC = \angle ABC = 180^o - 115^o = 65^o$

$CD^2 = 4^2 + 3^2 - 2(4)(3)\cos 65^o$

$CD = 3.854$

14. Let $f(x) = x + 2x^2 + 3x^3 + \cdots + 2nx^{2n}$

Remainder $= f(-1)$

$= \underbrace{-1 + 2 - 3 + 4 - 5 + \cdots + 2n}_{2n \text{ terms}}$

$= \underbrace{1 + 1 + 1 + \cdots + 1}_{n \text{ terms}}$

$= n$

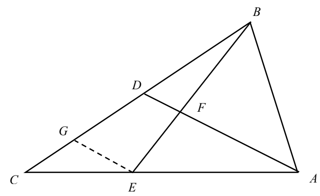
15. $\angle ABC = 108^o$

$\angle ACB = \angle CAB = 36^o$

$\angle AGB = 180^o - 36^o - 60^o$

$\angle AGB = 84^o$

16. Let G be a point on BC such that $EG//AD$



$\because CE : EA = 1 : 3$

$\therefore CG : GD = 1 : 3$

as AD is median,

$\therefore CD = DB$

$\Rightarrow CG : GD : DB = 1 : 3 : 4$

$\Rightarrow BF : FE = 4 : 3$

17. in ΔABD,

$\frac{AD}{\sin \angle ABD} = \frac{AB}{\sin \angle ADB}$

$\frac{\sin \angle ABD}{\sin \angle ADB} = \frac{AD}{AB}$

in ΔBCD,

$\frac{DC}{\sin \angle CBD} = \frac{BC}{\sin \angle BDC}$

$\frac{\sin \angle CBD}{\sin \angle BDC} = \frac{CD}{BC}$

$\because \angle ABD = \angle CBD$

and $\sin \angle BDC = \sin(180^o - \angle ADB)$

$= \sin \angle ADB$

$\therefore \frac{AD}{AB} = \frac{CD}{BC}$

$AB \times CD = BC \times AD$

18. $\dfrac{\sin^2(180°+\theta)+\cos^2\theta}{\tan(90°+\theta)\cos(180°+\theta)}$

$= \dfrac{\sin^2\theta+\cos^2\theta}{\frac{-1}{\tan\theta}(-\cos\theta)}$

$= \dfrac{\tan\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos^2\theta}$

19. let $AE = 1$, $EB = 2$ and $AD = 3$

$\tan\angle ADE = \frac{1}{3} \Rightarrow \angle ADE = 18.43^o$

$\angle EDG = 90^o - 18.43^o = 71.57^o$

$\angle EGD = 180^o - 2\times 71.57^o = 36.87^o$

$ED = \sqrt{3^2+1^2} = \sqrt{10}$

$ED^2 = DG^2 + EG^2 - 2(DG)(EG)\cos\angle EGD$

$10 = 2DG^2 - 2DG^2\cos 36.87^o$

$DG = 5$

$\therefore CG = 2 = EB$

$\Rightarrow \Delta BEF \cong \Delta CGF$

$\Rightarrow BF = CF = 1.5$

$\tan\angle BFE = \frac{2}{1.5} \Rightarrow \angle BFE = 53.13^o$

$\tan\angle CFD = \frac{3}{1.5} \Rightarrow \angle CFD = 63.43^o$

$\angle DFE = 180^o - 53.13^o - 63.43^o = 63.44^o$

20. $\Delta MBC \sim \Delta MAD$

$\frac{AM}{BM} = \frac{DM}{CM}$

$\frac{AM}{4} = \frac{5}{6}$

$AM = \frac{10}{3}$

21. I and II are obvious

by consider exterior angle,

$a + d = b + c$

$a + \frac{c}{2} = b + c$

$2a = c + 2b \quad \Rightarrow$ III is correct

22. $\dfrac{1+bi^3}{i}$

$= \dfrac{1+bi^3}{i}\cdot\dfrac{i}{i}$

$= \dfrac{i+bi^4}{-1}$

$= -(i+b)$

$= -b - i$

23. for $2x + y - 5 = 0$

x-intercept: $\frac{5}{2}$

y-intercept: 5

for $x - y - 3 = 0$

x-intercept: 3

y-intercept: -3

By choosing right hand side region for both equation, B is the answer

24. cost for each orange $= \frac{x}{N}$

selling price of each orange $= \frac{x}{N} + 3$

number of oranges sold $= N - 10$

$\therefore (N - 10)(\frac{x}{N} + 3) = x + 100$

25. suppose A take first, he has 3 choice

$3(1 + 2\times 1) = 9$

if the second choice is other than A, there are 2 choice, but 1 choice left for the last student

if the second student choose A's test paper, only 1 arrangement left for the rest

26. P(prouct is even)

$= 1 - P$(both odd)

$= 1 - \frac{2}{6}\times\frac{1}{5}$

$= \frac{14}{15}$

27. Join A and B, then $\angle ABO = \theta$

$\therefore \tan\theta = \frac{OA}{OB} = \frac{4}{6} = \frac{2}{3}$

28. lower quartile lie between 20th and 21st data

upper quartile lie between 60th and 61st data

IQR $= 2.5 - 0 = 2.5$

29. number of data cannot be observed from box-and-whisker diagram

30. for $y = \frac{2}{\sqrt{x-1}} + \sqrt{3-x}$ is define

$x - 1 > 0$ and $3 - x \geq 0$

$x > 1$ and $x \leq 3$

$1 < x \leq 3$

31. $\begin{cases} kx + y = 1 \\ y = x^2 \end{cases}$

$\Rightarrow kx + x^2 = 1$

$x^2 + kx - 1 = 0$

$\because$ x-coordinates of A and B are the roots of the above equation

$\therefore$ x-coordinates of the mid-pint of A and B are $-\frac{k}{2}$

sub into $kx + y = 1$

$y = 1 - k(-\frac{k}{2})$

$y = \frac{2+k^2}{2}$

32. slope of line $= -\frac{1}{2}$, y-intercept $= 1$

$\log_2 y = -\frac{1}{2}\log_2 x + 1$

$\log_2 y = \log_2 2x^{-\frac{1}{2}}$

$y = 2x^{-\frac{1}{2}}$

$y^2 = 4x^{-1}$

$xy^2 = 4$

33. $29\times16^8 + 4\times16^3 + 12\times16^2$

$= (16 + 13)\times16^8 + 4\times16^3 + 12\times16^2$

$= 16^9 + 13\times16^8 + 4\times16^3 + 12\times16^2$

$= 1D00004C00_{16}$

34. $\alpha^4 + \beta^4$

$= \alpha^4 + \beta^4 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2$

$= (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$= [(-\frac{b}{a})^2 - 2\frac{c}{a}]^2 - 2(\frac{c}{a})^2$

$= (\frac{b^2-2ac}{a^2})^2 - \frac{2c^2}{a^2}$

$= \frac{b^4-4ab^2c+4a^2c^2}{a^4} - \frac{2c^2}{a^2}$

$= \frac{b^4-4ab^2c+2a^2c^2}{a^4}$

35. half period $= 360^o \Rightarrow$ enlarged along y-axis by 2 times $\Rightarrow y = \sin\frac{x}{2}$

difference between midline and upper limit is 2 $\Rightarrow$ enlarged along y-axis by 2 times $\Rightarrow$ $y = 2\sin\frac{x}{2}$

y-intercept $= 3$ $\Rightarrow$translated upward by 3 units $\Rightarrow$ $y = 3 + 2\sin\frac{x}{2}$

36. $\log_2(x + 1) - \log_2(2x + 1) < 0$

$\log_2\left(\frac{x+1}{2x+1}\right) < 0$

$0 < \frac{x+1}{2x+1} < 1$

for $0 < \frac{x+1}{2x+1}$

$x < -1$(rej) or $x > -\frac{1}{2}$

for $\frac{x+1}{2x+1} < 1$

$(x + 1)(2x + 1) < (2x + 1)^2$

$0 < x(2x + 1)$

$x < -\frac{1}{2}$(rej) or $x > 0$

$\therefore x > -\frac{1}{2}$ and $x > 0$

$\Rightarrow X > 0$

37. join BC and AD

$\because \Delta VBC \sim \Delta VDA$

$\therefore \frac{VB}{VD} = \frac{VC}{VA}$

$\frac{12}{16+CD} = \frac{16}{24}$

$CD = 2$

38. I and III are obvious

for II, if $0 < a < 1$, $g(x)$ will be reduced along y-axis from $f(x)$

$\Rightarrow$ II is incorrect

39. let $P(x, y)$ be the locus

$\because P$ is the mid-point of AB

$\therefore A = (2x, 0)$ and $A = (0,2y)$

as $AB = 10$

$\therefore (2x)^2 + (2y)^2 = 10^2$

$x^2 + y^2 = 25$

40. area of $\Delta ABC = \sqrt{3}$

$\frac{1}{2}AB^2\sin 60^o = \sqrt{3}$

$AB = 2$

height of a regular tetrahedron

$= \sqrt{\frac{2}{3}}\times$length of side

$\therefore$ height $= 2\times\sqrt{\frac{2}{3}} = \frac{2\sqrt{6}}{3}$

$\Rightarrow$ Volume $= \frac{1}{3}\times\sqrt{3}\times\frac{2\sqrt{6}}{3}$

$= \frac{2\sqrt{2}}{3}$

41. $CB = CH$
$\therefore$ I is correct

II and III are obvious incorrect

42. let $M(h, 0)$ be the required coordinates
$MA = MC$
$\sqrt{h^2 + 3^2} = 4 - h$
$h = \frac{7}{8}$

43. let $\angle AOB = x$, $\angle BOC = 2x$, $\angle BDC = 3x$
$\because OC$ and OB are tangent,
$\therefore \angle BCO = \angle CBO = \angle BDC = 3x$
$\Longrightarrow 2x + 3x + 3x = 180^o$
$x = 22.5^o$
$\because OC = OB$ and $OB = OA$
$\therefore OC = OA$
$\Longrightarrow \angle OCA = 56.25^o$
$\angle ACB = \angle OCB - \angle OCA$
$= 67.5^o - 56.25^o$
$= 11.25^o$

44. II is obvious
a, b or c can be negative, II is incorrect

for $ax^2 + 2bx + c = 0$ has double roots
$(2b)^2 - 4ac = 0$
$b^2 = ac$
$\frac{b}{a} = \frac{c}{b}$ $\Longrightarrow$ III is correct

45. ${\sigma_A}^2 = \frac{\sum(x_A - \overline{x})^2}{n}$
$\Longrightarrow 36\times 8 = \sum(x_A - \overline{x})^2$
$\sum(x_A - 168)^2 = 288$

similarly,
${\sigma_B}^2 = \frac{\sum(x_B - \overline{x})^2}{n}$
$\sum(x_B - 168)^2 = 48\times 6$
$= 288$

${\sigma_{A+B}}^2 = \frac{\sum(x_A - 168)^2 + \sum(x_B - 168)^2}{14}$
${\sigma_{A+B}}^2 = \frac{288+288}{14}$
${\sigma_{A+B}}^2 = 41.1$